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VOLUME III

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STUDIES IN THE METHODOLOGY OF WEAPONS SYSTEMS EFFECTIVENESS ANALYSIS—USING THE TECHNIQUES OF SIMULATION, OPTIMIZATION AND STATISTICS—PHASE II

VOLUME III. EFFECTS OF PARAMETER VARIATIONS ON THE CAPABILITY OF A PROPORTIONAL NAVIGATION MISSILE AGAINST AN OPTIMALLY EVADING TARGET IN THE HORIZONTAL PLANE

DEPARTMENT OF ELECTRICAL ENGINEERING
LOUISIANA STATE UNIVERSITY

DDC

OCT 14 1970

AIR FORCE ARMAMENT LABORATORY

AIR FORCE SYSTEMS COMMAND • UNITED STATES AIR FORCE

EGLIN AIR FORCE BASE, FLORIDA

STUDIES IN THE METHODOLOGY OF WEAPONS SYSTEMS EFFECTIVENESS
ANALYSIS - USING THE TECHNIQUES OF SIMULATION, OPTIMIZATION
AND STATISTICS - PHASE II

Volume III. Effects of Parameter Variations on the Capability
of a Proportional Navigation Missile Against an
Optimally Evading Target in the Horizontal Plane

Paul M. Julich

David A. Borg

FOREWORD

This report covers work done during the period September 1968 through August 1969 by Louisiana State University, Baton Rouge, Louisiana, under contract F08635-68-C-0107 with the Air Force Armament Laboratory, Eglin Air Force Base, Florida. Program monitor for the Armament Laboratory was Lt. Jerry L. Edwards (ATAD). Project Director for Louisiana State University was Dr. Adrain E. Johnson, Jr., Department of Chemical Engineering.

The report consists of four volumes as follows: Volume I - MOD6DF Systems Simulation, Volume II - Missile Simulation, Volume III - Effects of Parameter Variations on the Capability of a Proportional Navigation Missile Against an Optimally Evading Target in the Horizontal Plane, and Volume IV - Formulation and Optimization of Warhead Kill Probabilities.

Information in this report is embargoed under the Department of State International Traffic In Arms Regulations. This report may be released to foreign governments by departments or agencies of the U. S. Government subject to approval of the Air Force Armament Laboratory (ATAD), Eglin AFB, Florida 32542, or higher authority within the Department of State export license.

This technical report has been reviewed and is approved.

Thomas P Christie

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Chief, Analysis Division

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ABSTRACT

The problem considered is a two-dimensional, constant velocity, point model of a target in an encounter with a proportional navigation pursuer. The horizontal plane is chosen so that the effects of gravity may be neglected. Constraints on the turning rate and time delays in both the pursuer's and target's guidance system are included.

The sensitivity of the miss distance to variations in the parameters associated with pursuer and evader is presented.

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SECTION I

INTRODUCTION

With the introduction of modern control theory, interest has been stimulated in optimal pursuit-evasion strategies. From studies of differential games it has been indicated that for certain formulations proportional navigation constitutes the optimal pursuit strategy [1].

A two-dimensional, constant velocity, point model of a target in an encounter with a proportional navigation pursuer is considered. The horizontal plane is chosen so that effects of gravity may be neglected. Mechanization of proportional navigation introduces numerous effects not included in the differential game formulation [1]. Three of the most important of these are limits on the turning capability of the pursuer, a time lag in the control system of the pursuer, and a time lag in the target's control system. Previous studies [2] determined the optimum evasive tactics of a model including the first two of these effects. The effects of variations of target and pursuer parameters on the optimum miss and the optimum target control are discussed.

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SECTION II

DISCUSSION

Mathematical Description of Problem

Figure 1 gives a pictorial description of the encounter. A set of state variables s, y, ϕ_T, and ϕ_P were chosen and result in the following differential equations.

$$\dot{x} = V_T \cos \phi_T - V_P \cos \phi_P \tag{II-1}$$

$$\dot{y} = V_T \sin \phi_T - V_P \sin \phi_P \tag{II-2}$$

$$\dot{\psi} = U_P \tag{II-3}$$

$$\dot{\phi}_T = U_T \tag{II-4}$$

The turning rate of the pursuer, U_p, is determined by the proportional navigation system [3]. In ideal proportional navigation the turning rate of the pursuer is proportional to the time rate of change of the line of sight, $\dot{\phi}$, from the pursuer to the target. However, for this problem two important limiting effects on the pursuer are included. These are (1) a limit on the turning radius of the pursuer and (2) a time lag in the pursuer's guidance system.

Figure 2 shows a block diagram of the simplified guidance system for two constraints mentioned above. A nonlinear function $\eta(a\dot{\phi})$ limits the turning radius of the pursuer. It was assumed that hard limiting on the number of G's the pursuer may withstand was desired. However, the application of hard limits caused discontinuities in the derivatives needed for the optimization method used, so an approximation was made. The arctangent function, Figure 3, was chosen to approximate the hard limits. An additional differential equation is determined from Figure 2 and 3 which governs the behavior of the pursuer's turning rate, U_P.

$$\dot{U}_P = \frac{1}{\tau_P} U_{MAXP} \ (3/\pi) \ \tan^{-1}\left(\sqrt{3} \ \frac{a\dot{\phi}}{U_{MAXP}}\right) - \frac{1}{\tau_P} \tag{II-5}$$

where

$$\dot{\phi} = \frac{\dot{y}x - y\dot{x}}{x^2 + y^2} \qquad U_{MAXP} = \frac{G_{MAXP}(32.2)}{V_P} \ ,$$

τ_P is the time constant introduced into the pursuer's guidance system, a is the proportional navigation constant, and G_{MAXP} is the desired limit on the pursuer G's.

The same types of limitations imposed on the pursuer are assumed to be
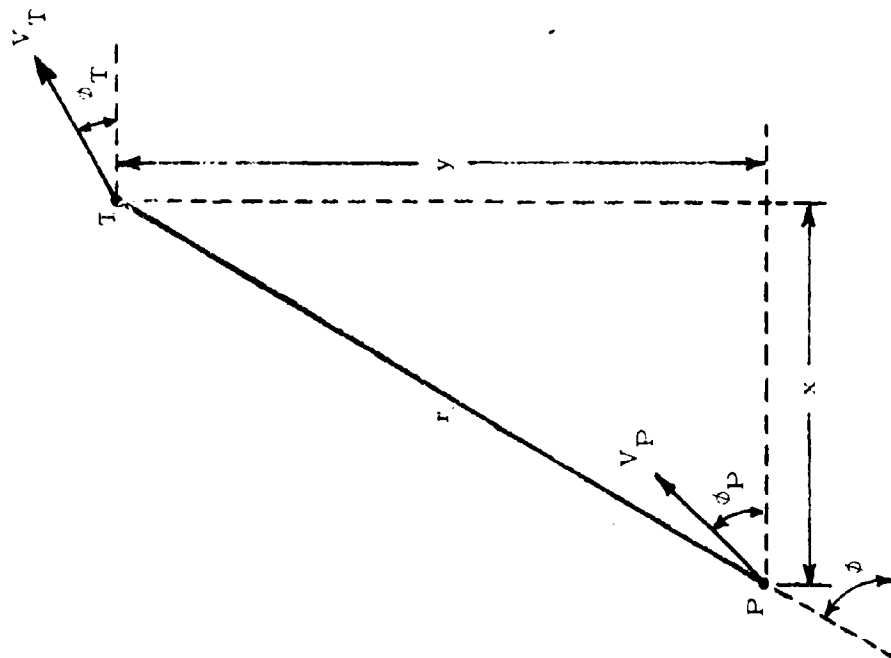
V_T - target velocity

ϑ_T - angle of target velocity with the abscissa

V_P - pursuer velocity

ϑ_P - angle of pursuer velocity

ϕ - angle of the line-of-sight from the pursuer to the target with the abscissa

r - the distance between the target and the pursuer

x - distance along the abscissa

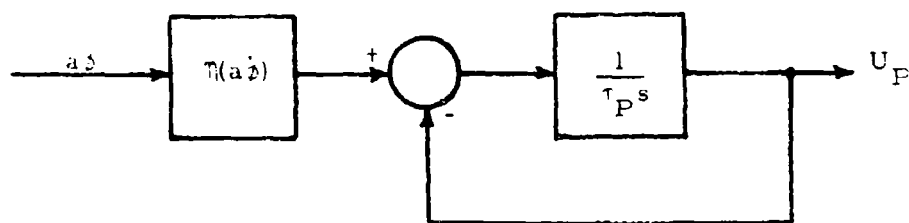y - distance along the ordinate

Figure 1. Problem Geometry.

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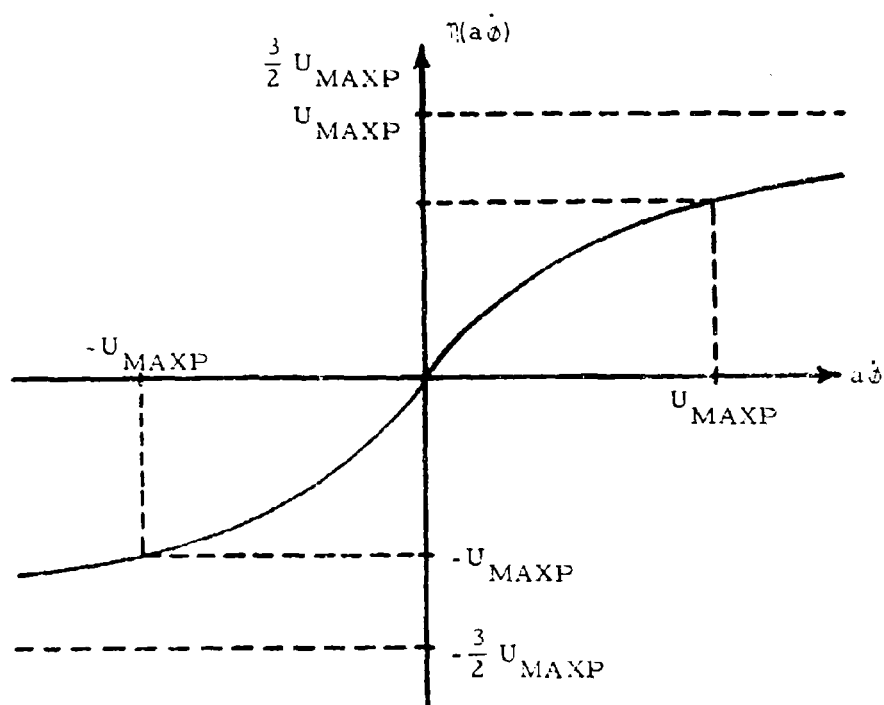Figure 2. Block Diagram of Pursuer's Guidance System.

Figure 3. Plot of $\eta(a\dot{\phi}) = U_{MAXP}\left(\frac{3}{\pi}\right)\tan^{-1}\left(\sqrt{3}\,\frac{a\dot{\phi}}{U_{MAXP}}\right)$

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inherent in the target. Figure 4 is a block diagram of the target control system. Using Figure 4 and imposing a hard limit on the target control, u, the following differential equation is written.

$$\dot{U}_T = \frac{1}{\tau_T} U_{MAXT} u - \frac{1}{\tau_T} U_T \; ; \; |u| \le 1 \qquad (II\text{-}6)$$

where

$$U_{MAXT} = \frac{G_{MAXT}(32.2)}{V_T} \; ,$$

τ_T is the time constant introduced into the target's control system, and G_{MAXT} is the desired limit on the target G's.
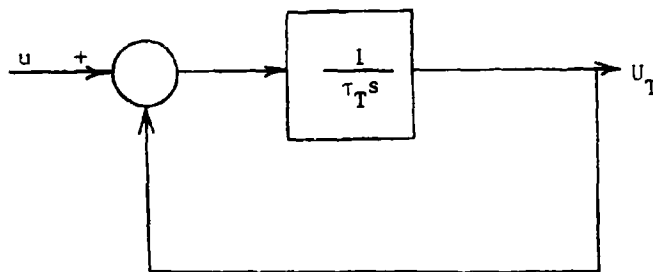
Figure 4. Block Diagram of Target's Control System

The goal of the target's maneuvers is to obtain the greatest possible miss by the pursuer. It is assumed that once the pursuer passes the point of closest approach to the target the pursuer is unable to turn and complete a second attack on the target. Hence, an appropriate measure of the effectiveness of the target evasive action is given by:

$$J = -r^2(t_f) = - x^2(t_f) - y^2(t_f) \qquad (II\text{-}7)$$

where t_f is the effective terminal time of the problem when the pursuer has reached its closest approach to the target or

$$\dot{r}(t_f) = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} = 0 .$$

Optimum Control Problem

An optimal control problem can now be formulated. Given the plant equations (II-1) through (II-6) and the index of performance (II-7), choose $u \in V$, $V = \{u: |u| \le 1\}$ so that $-r^2(t_f)$ is a minimum, subject to the differential side constraint (Equation (II-1) through (II-6))

Solution of Optimum Control Problem

Using Pontryagin's Maximum Principle 4 the following control Hamiltonian results:

$$H = p_x (V_T \cos \phi_T - V_P \cos \phi_P) + p_y (V_T \sin \phi_T - V_P \sin \phi_P)$$

$$+ p_{\phi_P} U_P + p_{\phi_T} U_T + p_{U_P} \left[\frac{1}{\tau_P} U_{MAXP} (3/\pi) \tan^{-1} \left(\sqrt{3} \ \frac{a\phi}{\ddot{u}_{MAXP}} \right) \right.$$

$$\left. - \frac{1}{\tau_P} U_P \right] + p_{U_T} \left(\frac{1}{\tau_T} U_{MAXT} u - \frac{1}{\tau_T} U_T \right)$$

Note that the target control, u, enters the Hamiltonian linearly. Thus

$$u^*(t) = sgn \left[p_{U_T} (t) \right] \tag{II-8}$$

The costate equations must satisfy the differential equations:

$$\dot{p}_x = -\frac{\partial H}{\partial x} = p_{U_P} \frac{AC}{1 + (A\dot{\phi})^2} \left[\frac{-\dot{y}(x^2 + y^2) + 2(\dot{y}x - y\dot{x})x}{(x^2 + y^2)^2} \right] \tag{II-9}$$

$$\dot{p}_y = -\frac{\partial H}{\partial y} = p_{U_P} \frac{AC}{1 + (A\dot{\phi})^2} \left[\frac{\dot{x}(x^2 + y^2) + 2(\dot{y}x - y\dot{x})y}{(x^2 + y^2)^2} \right] \tag{II-10}$$

$$\dot{p}_{\phi_P} = -\frac{\partial H}{\partial \phi_P} = -p_x V_P \sin \phi_P + p_y V_P \cos \phi_P$$

$$+ p_{U_P} \frac{AC}{1 + (A\dot{\phi})^2} \left[\frac{-x(V_P \cos \phi_P) + y(V_P \sin \phi_P)}{x^2 + y^2} \right] \tag{II-11}$$

$$\dot{p}_{\phi_T} = -\frac{\partial H}{\partial \phi_T} = p_x V_T \sin \phi_T - p_y V_T \cos \phi_T$$

$$-p_{U_P} \frac{AC}{1 + (A\dot{\phi})^2} \left[\frac{-x(V_T \cos \phi_T) + y(V_T \sin \phi_T)}{x^2 + y^2} \right] \tag{II-12}$$

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$$\dot{P}_{U_P} = -\frac{\partial H}{\partial U_P} = -P_{\phi_P} + B_{P_{U_P}} \tag{II-13}$$

$$\dot{P}_{U_T} = -\frac{\partial H}{\partial U_T} = -P_{\phi_T} + D_{P_{U_T}} \tag{II-14}$$

where

$$A = \frac{\sqrt{3}}{U_{MAXP}} \, a$$

$$B = \frac{1}{\tau_P}$$

$$C = \frac{U_{MAXP}}{\tau_P} \, (3/\pi)$$

$$D = \frac{1}{\tau_T}$$

and

$$\dot{\phi} = \frac{\dot{y}x - y\dot{x}}{x^2 + y^2}$$

A two-point boundary value problem results with the following boundary conditions:

$$x(0), \ y(0), \ \phi_P(0), \ \phi_T(0), \ U_P(0), \ \text{and} \ U_T(0) \tag{II-15}$$

given and from the transversality conditions

$$P_x(t_f) = 2x(t_f)$$

$$P_y(t_f) = 2y(t_f)$$

$$P_{\phi_P}(t_f) = 0$$

$$P_{\phi_T}(t_f) = 0 \tag{II-16}$$

$$P_{U_P}(t_f) = 0$$

$$P_{U_T}(t_f) = 0$$

where t_f is the terminal time.

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Choice of Initial Conditions

In Equation (II-15) it is stated that the initial conditions on the state variables are given. This allows six-degrees-of-freedom in the specification of the problem. By noting that it should always be possible to choose the axis used to write the system differential equations, see Figure 1, such that the x-axis is parallel to the initial velocity vector of the target without effectively changing the results, $\phi_T(0)$ may be specified as zero without any loss of generality.

It is assumed that the pursuer is aimed such that if neither the target nor the pursuer applied any control interception would occur. This being the case, both the target and pursuer would follow a straight line course and from the geometry of the problem two of the initial conditions are specified in terms of other problem constants as follows:

$$u_p(0) = 0$$

$$\phi_p(0) = \phi(0) \sin^{-1}\left\{(V_T/V_p) \sin \phi(0)\right\}$$

where $\phi(0)$ is the initial angle of the line of sight from the pursuer to the target with the chosen abscissa. In order to further reduce the freedom in the specification of the problem it is assumed that $U_T(0) = 0$.

With the above considerations, only two initial conditions remain unspecified, $x(0)$ and $y(0)$. For convenience the initial range, $r(0)$, and the initial angle of the line of sight from the missile to the target with the abscissa, $\phi(0)$, are specified and $x(0)$ and $y(0)$ are calculated from these values.

Numerical Results

A steepest ascent procedure [2,5] for which the differential equations were numerically integrated using a 4th order Runge-Kutta method [6] with time step sizes of 0.1 seconds was formulated which would determine the optimal control. To assure good accuracy in determining the miss distance a method was used to reduce the time step size near the terminal time. The problem was programmed and run on an IBM 350/65 system. A nominal case where

$$V_T = 1013 \text{ ft/sec}$$

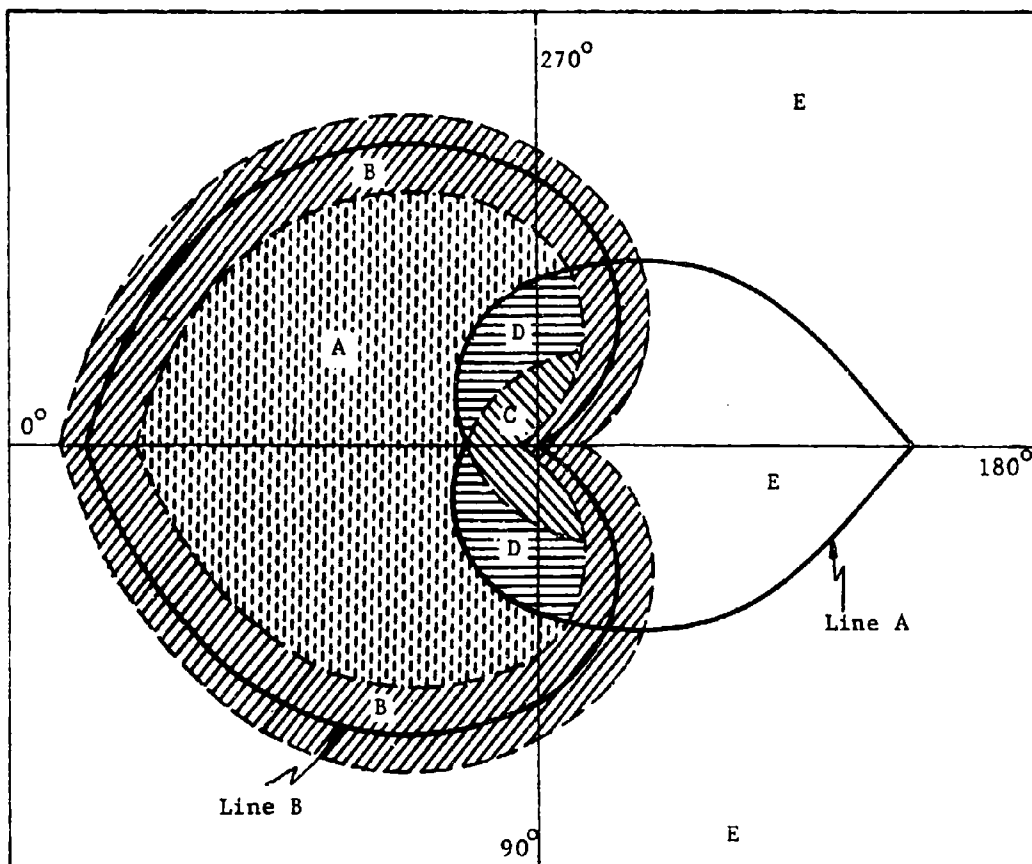$$V_P = 2252 \text{ ft/sec}$$

$$C_{MAXT} = 5$$

$$G_{MAXP} = 10$$

$$\tau_T = 1.0 \text{ sec}$$

$$\tau_P = 0.5 \text{ sec}$$

$$a = 3.0$$

was chosen and for particular choices or $r(0)$ and $\phi(0)$ some of the target and missile parameters were varied to determine their effect on the optimal target control and the terminal miss distance.

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Control sequences given for $0^o \leq \phi(0) \leq 180^o$. Control sequences for $180^o \leq \phi(0) \leq 360^o$ negative of the ones given.

- Area A - target maneuvers pursuer such that pursuer is directly behind target then executes optimal terminal maneuver. Maximum terminal miss approximately equivalent for all points within this area.

- Area B - Optimal control $u_T^*(t) = -1$ for all t, target able to turn inside of pursuer trajectory.

- Area C - Optimal control bang-bang with one switch, control sequence $(+1,-1)$.

- Area D - Optimal control bang-bang with two switches, control sequence $(+1,-1,+1)$.

- Area E - Probable control logic is to maneuver pursuer such that control logic of Area B may be followed.

Figure 5. Target Control Sequence Versus Pursuer Initial Position.

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Previous work [2] has shown that in order for the target's maneuver to obtain the greatest miss the target must choose, depending on the initial conditions $r(0)$ and $\phi(0)$, the best from several control philosophies. The control philosophies thus far uncovered result in one of the following types of control:

Control A: The optimal control contains two or more switches and only the terminal portion of the control has converged to the constraint ($|U(t)| = 1$) boundary.

Control B: The optimal control is $U(t) = +1$ (or $U(t) = -1$) for all $t\epsilon(0, t_f)$.

Control C: The optimal control switches once from $U(t) = +1$ to $U(t) = -1$ (or switches once from $U(t) = -1$ to $U(t) = +1$).

Control D: The optimal control contains two switches from one side of the constraint boundary to the other.

Figure 5 presents these controls with their probable philosophy as a function of the pursuer's initial position [2]. The shape of the control logic areas in this figure are only a "best guess" estimate of the actual shape from data presently available. The distance from the origin is $r(0)$, whereas the angle measured in a counter-clockwise direction from the line marked $0°$ is $\phi(0)$. The controls thus far determined resulting from initial conditions in area E are of the form of Control C or Control D.

For Figures 6 through 10 one of the target's or pursuer's parameters is varied (varied parameter noted along the abscissa), while the other parameters are maintained at the values denoted in Equation II-17. The (a) part of each figure gives the miss distance obtained when the target uses the optimal control as a function of the varied parameter. The (b) part of each figure presents the difference between the terminal time, t_f, and the last switching time, t_n, and the difference between the last switching time, t_n and the next to last switching time, t_{n-1}, (if these differences exist) of the optimal control as a function of the target's or pursuer's varied parameter. The difference $t_f - t_n$ and $t_n - t_{n-1}$ will be denoted by "difference in switching time" in future reference. Unless otherwise noted, it is to be assumed that for each of the figures the initial range is 10,000 feet and the initial angle of the line of sight is 0 degrees.

In Figure 6 the terminal miss and the difference in switching times is plotted versus target and pursuer velocity. The solid lines give the variations with changes in pursuer speed when the target speed is constant at 1,013 ft/sec and the dotted line denotes variations with changes in the target's speed when the pursuer's speed is held constant at 2,252 ft/sec. There appears to be a greatest miss as well as a least miss for the pursuer's speed within the range shown. For pursuer velocities of 4,116 ft/sec and 3,805 ft/sec the optimal control was of the type of Control C and for the points run where the pursuer's velocity was between 3,494 ft/sec and 2,561 ft/sec the optimal control was of the form of Control D. For pursuer

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TARGET'S SPEED (in ft/sec)

394   703   1013   1323   1633   1942

30—

25—

MISS DISTANCE (in feet)

Vary V_T

Vary V_P

20—

15—

10—

5—

0—

4116   3805   3494   3183   2872   2561   2252   1942   1633   1323
              2716   2407   2097   1788   1478

PURSUER'S SPEED (in ft/sec) (a)

TARGET'S SPEED (in ft/sec)

394   703   1013   1323   1633   1942

3.0—

Vary V_T

2.5—

TIME (in seconds)

2.0—

Vary V_P

$t_f - t_n$

1.5—

$t_n - t_{n-1}$

1.0—

.0—

4116   3805   3494   3183   2872   2561   2252   1942   1633   1323
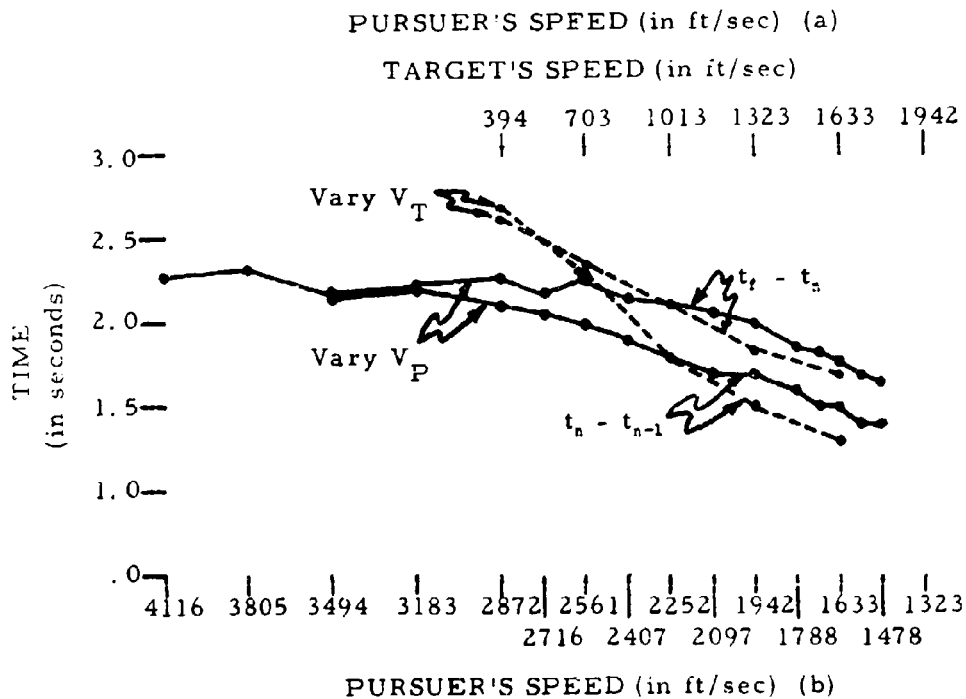   2716   2407   2097   1788   1478

PURSUER'S SPEED (in ft/sec) (b)

Figure 6.   Effects of Varying the Target's and Pursuer
Velocity. ($r(0) = 10,000$ ft. and $\phi(0) = 0.0°$)

11

30 —

25 —

@ 5G's $r(t_f)$ = 266 ft.

MISS DISTANCE (in feet)

20 —

15 —

10 —

5 —

0 ┐

0   5   10   15   20   25   30   35   40

G LIMIT ON PURSUER

(a)

2.5 —

2.0 —

TIME (in seconds)

1.5 —

$t_f - t_n$

1.0 —

$t_n - t_{n-1}$

.5 —

0 ┐

0   5   10   15   20   25   30   35   40

G LIMIT ON PURSUER

(b)
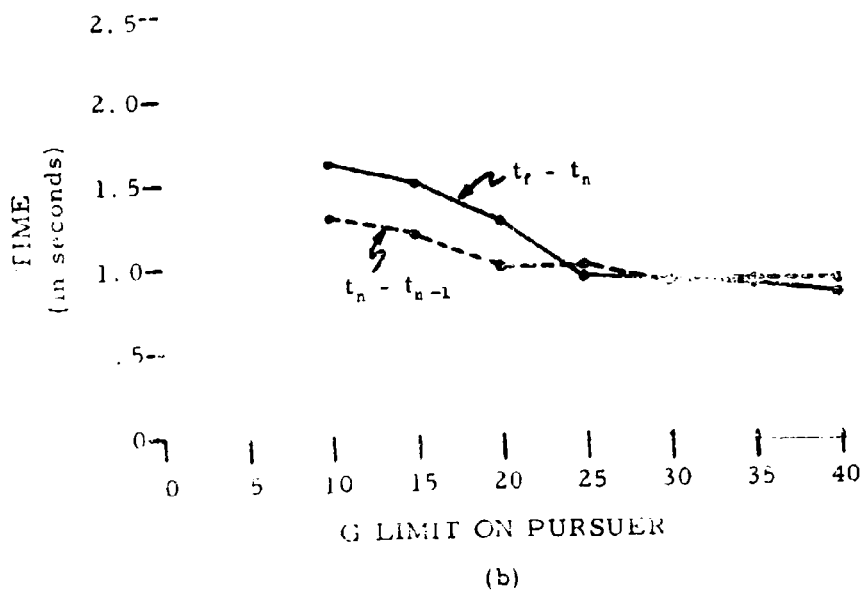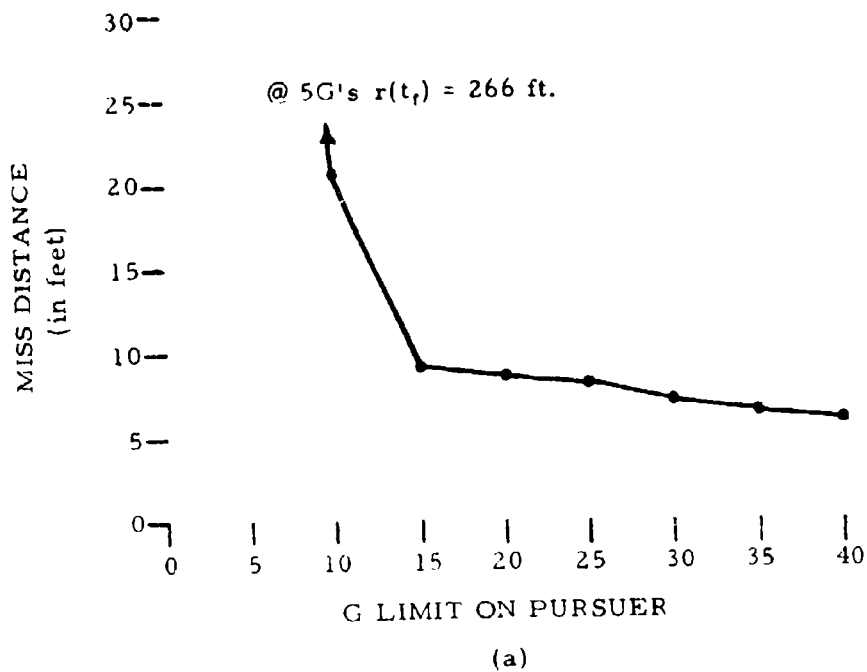
Figure 7. Effects of Varying the Pursuer's G Limit.
($r(0)$ = 10,000 ft. and $z(0)$ = 0.0°)

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MISS DISTANCE (in feet)

PROPORTIONAL NAVIGATION CONSTANT (a)

TIME (in seconds)

$t_f - t_n$

$t_n - t_{n-1}$

PROPORTIONAL NAVIGATION CONSTANT (b)

Figure 8.   Effects of Varying the Proportional Navigation
Constant.   ($r(0) = 10,000$ ft. and $\dot{\gamma}(0) = 0.0^\circ$)

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(a)

(b)

Figure 9. Effects of Varying the Pursuer's Time
Constant. $(r(0) = 10,000$ ft. and $\sigma(0) = 0.0°)$

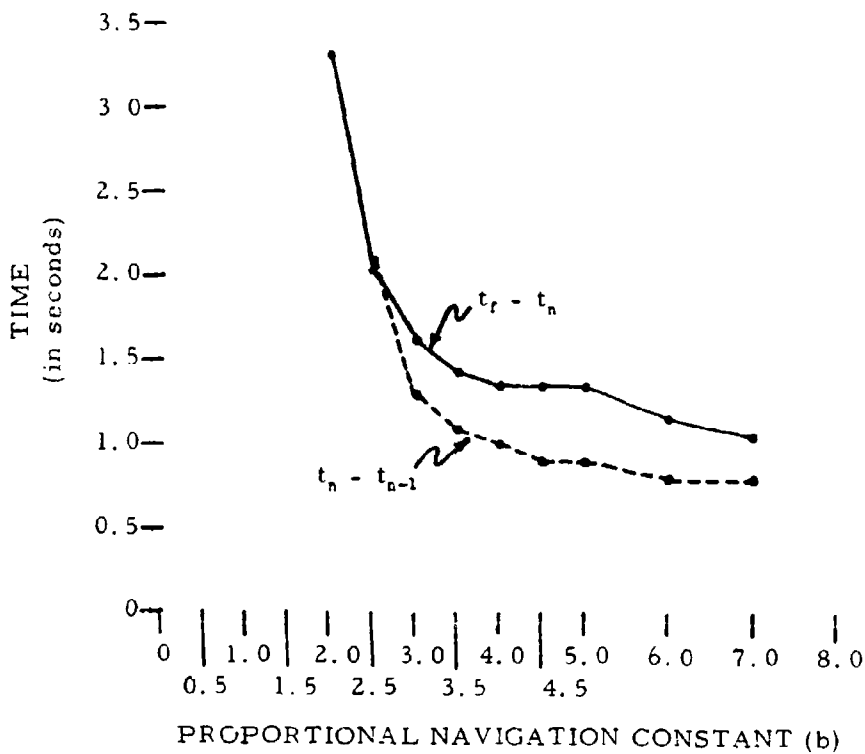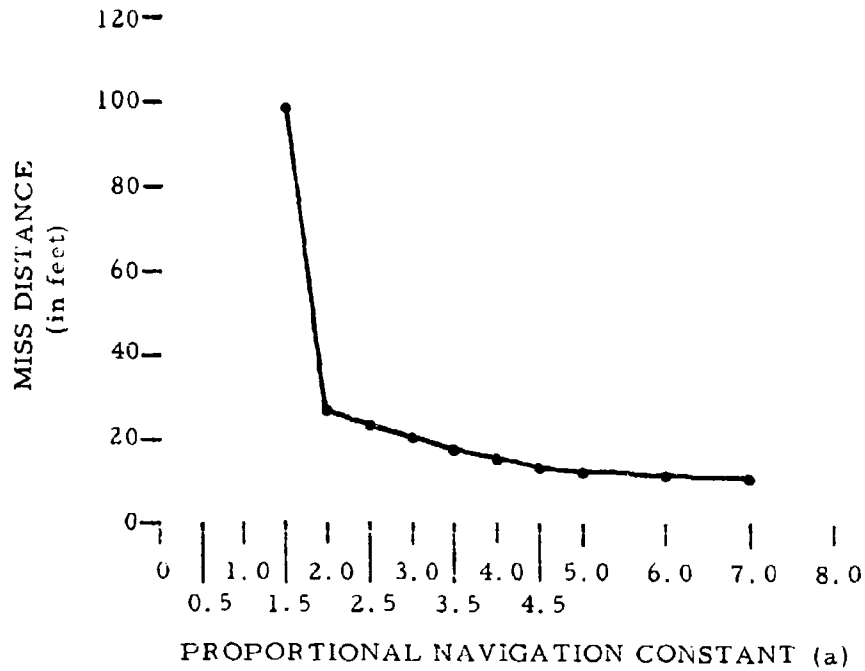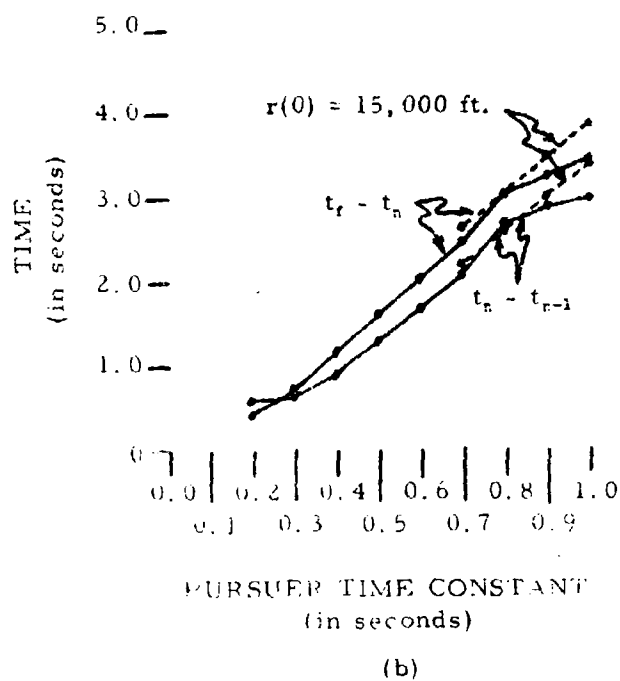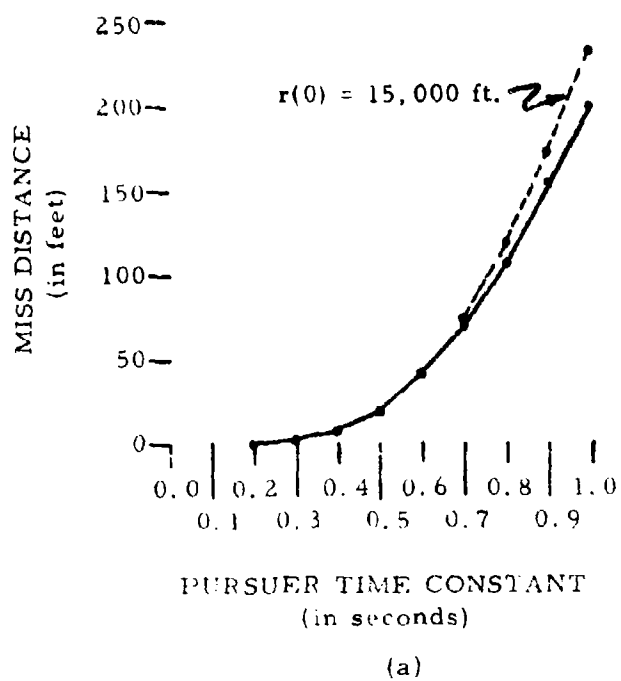speeds between 2,407 ft/sec and 1,478 ft/sec the best target control calcu-
lated was like Control A. It is interesting to note that local minimums were
uncovered where the target control was $U(t) = -1$ for all t_ϵ $!0$, $t_f]$, Control
B, for pursuer velocities of 1,478 ft/sec and 1,400 ft/sec and that the
optimal control for the pursuer speed of 1,323 ft/sec was of the same type.
This information is presented by the crosshatched line in Figure 6a and
shows that the target control logic changes as the pursuer's speed is varied
between 1,478 ft/sec and 1,322 ft/sec. For a target speed of 394 ft/sec the
optimal control was of the form of Control D and for the other points tried
the optimal control was of the type of Control A.

Figure 7 is a plot of the miss distance and the difference in switch-
ing times as a function of the G limit imposed on the pursuer. As might be
expected the miss distance decreases as the allowable number of G's the
pursuer may withstand is increased. However, it appears that the improve-
ment in the pursuer's ability to capture the target is small if the G limits
are increased beyond (for this case) 15 G . The difference in switching
times apparently decreases as the G limit imposed on the pursuer is in-
creased. The form of the optimal control for the case where the pursuer's
G limit was the same as the target's (or 5 G's) was of the type of Control
B. For all other points calculated the optimal control was of the type of
Control A.

Figure 8 gives the effects of varying the proportional navigation con-
stant. For the problem formulated both the miss distance and the difference
in switching times decreases as the proportional navigation constant is in-
creased. The optimum control for a proportional navigation constant of 1.5
was of the form of Control B, while proportional navigation constant of 2.0
resulted in an optimal control of the type of Control C. The optimal con-
trol for all other calculated points was of the form of Control A.

Figure 9 demonstrates the effects of varying the pursuer time constant.
The solid lines in the a and b parts of the figure represent information cal-
culated for an $r(0)$ of 10,000 feet and $c(0)$ of 0 degrees while the dotted
line depict information obtained for an $r(0)$ of 15,000 feet and $\phi(0)$ of 0
degrees. Both the miss distance and difference in switching times increases
as the delay in the pursuers guidance system is increased. The form of the
optimal control for $r(0) = 10,000$ feet was of Control A for missile time de-
lays of 0.2 sec. through 0.7 sec. and of Control D for the remaining points
calculated. The $r(0) = 15,000$ feet case was presented because it appears
that the limit imposed by the terminal time for the case of $r(0) = 10,000$
feet limited the freedom of the switching times to vary. All the points
calculated for $r(0) = 15,000$ feet resulted in an optimal control of the
form of Control A.

The last figure, Figure 10, represents information obtained when the
target or pursuer speed is varied for the case when $r(0) = 20,000$ feet and
$c(0) = 180$ degrees. For this case, head-on launch, the miss distance and
terminal difference in switching time both decrease as the pursuer's speed
is increased (solid line), while the target's speed is constant at 1,013
ft/sec. If the pursuer's speed is held constant at 2,252 ft/sec and the
target's speed is increased from 394 ft/sec, the miss distance and the ter-
minal switching time are increased. Again there is noted a change in target

TARGET'S SPEED (in ft/sec)

394   703   1013 1323 1623 1942

(a)

TARGET'S SPEED (in ft/sec)

394   703 1013 1323 1633 1942

(b)

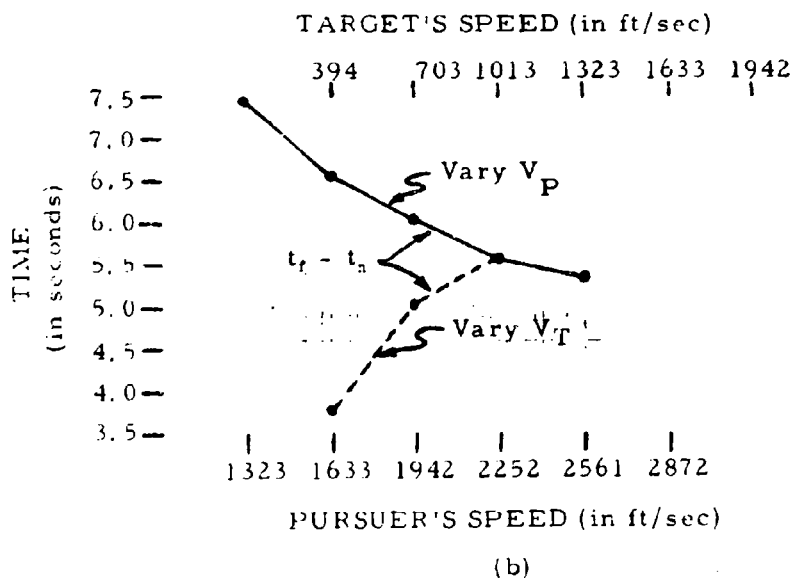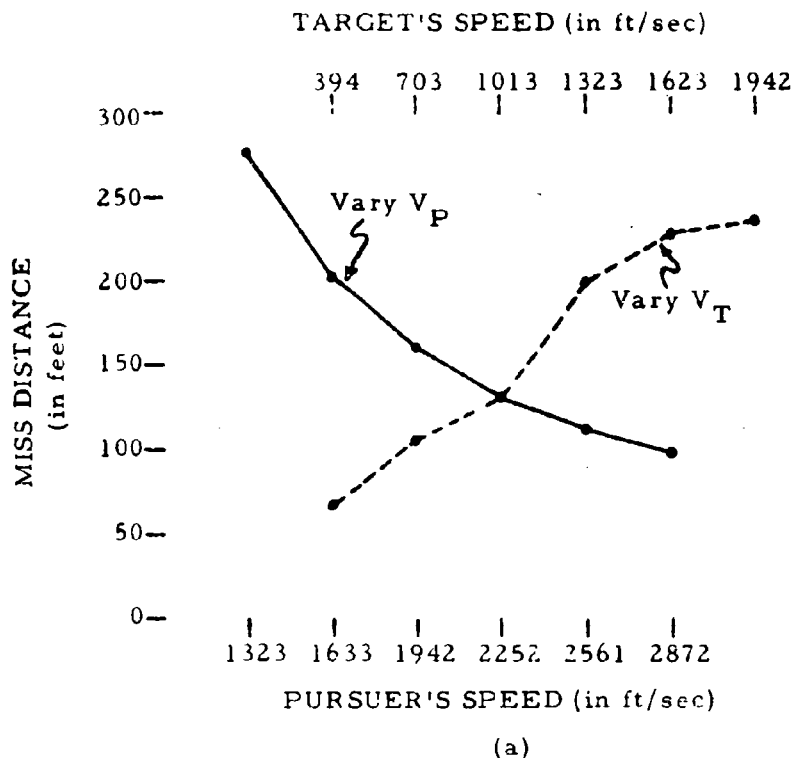Figure 10.   Effects of Varying Target's and Pursuer's Velocity -
The Head - on Case. (r(0) = 20,000 ft. and :(0) = 180.0°).

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control logic as the target's or pursuer's speed is changed through the range shown. However, it should be noted that the control logic resulting in control of the type of Control B, C and D is different for the head-on launch from the logic resulting in these controls when $\phi(0) = 0^o$. For target speed of 394 ft/sec the optimal control is of the form of Control D; for target speed of 703 ft/sec and 1,013 ft/sec the form of the control is of Control C; and for target speeds of 1,323 ft/sec, 1,633 ft/sec, and 1,942 ft/sec the form of the control is control B. For pursuer speeds of 1,323 ft/sec through 2,561 ft/sec the control is similar to control C, while a pursuer speed of 2,872 ft/sec resulted in a control like Control B.

The results of the parameter sensitivity studies indicate that, as expected, the miss distance is strongly dependent on the parameters of the proportional navigation system. The results follow the trend described in [7], where a proportional navigation missile is studied against a target employing fixed (non-optimal) tactics.

An interesting effect is the differences observed for a launch from $\phi = 0^o$ and $\phi = 180^o$. The variations with velocity show a reversal for the cases $\phi = 0^o$, and $\phi = 180^o$. Thus, it is indicated that a low speed pursuer is preferable for rear hemisphere launches, while a high speed pursuer functions best for head-on launches. It is to be noted, however, that the preference of a high speed missile for head-on launches is based on the assumption of errorless launches. The high speed missile would be much more sensitive to launch errors. The launch error problem is under investigation and will be reported on in the near future.

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SECTION III

CONCLUSIONS

The results of this study indicate that best control logic for the target control and the choice of switching times for the optimal target control depend on the particular value of target and missile parameters, as well as the particular initial condition $r(0)$ and $\phi(0)$. The relatively smooth variations in the difference in switching times may make it possible to determine an empirical relationship between the switching times and terminal time, at least for some of the control philosophies, involving the target's and pursuer's parameters. In order to determine such a relationship it would probably be necessary to limit results to one control logic discipline. Additional work would then be necessary to determine the best control logic to follow.

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13. ABSTRACT

The problem considered is a two-dimensional, constant velocity, point model of a target in an encounter with a proportional navigation pursuer. The horizontal plane is chosen so that the effects of gravity may be neglected. Constraints on the turning rate and time delays in both the pursuer's and target's guidance system are included.

The sensitivity of the miss distance to variations in the parameters associated with pursuer and evader is presented.

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	ROLE	WT	ROLE	WT	ROLE	WT
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Pursuer						
Horizontal Plane						
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Proportional Navigation						
Evasion						
Optimization						